Progressive Education Society's
Modern College of Arts, Science & Commerce Ganeshkhind,
Pune – 16(Autonomous)
End Semester Examination : October/November 2024
Faculty: Science and Technology (2024-2025)

Program : BScGen03	**Semester: V**	**SET : A**
Program (Specific) : BSc (Regular)		**Course Type: DSEC/DSC**
Class: T.Y.B.Sc. (Mathematics)		**Max. Marks : 35**
Name of the Course : Real Analysis – I		**Course Code : 24-MT-352**
Paper No. : II		**Time : 2 Hrs.**

Instructions to the Candidates :

1)There are 3 sections in the question paper. Write each section on separate page.
2)All Sections are compulsory.
3)Figures to the right indicate full marks.
4)Draw a well labelled diagram whenever necessary.

SECTION-A

Q.1)Attempt any five of the following. **[Marks 10]**

a) Define i) Tautology ii) Contradiction.

b) Find the truth value of the statement $\forall x\ \exists y\ ,\ y < x^2$.

c) Show that card $(\mathbb{Z})$ = card $(\mathbb{N})$.

d) Define Limit of a sequence.

e) Show that $\left\{\sqrt{n+1} - \sqrt{n}\ \right\}_{n=1}^{n=\infty}$ is convergent.

f) Show that the sequence $\left\{\frac{1}{1+n^2}\right\}_{n=1}^{n=\infty}$ is monotonic.

g) Define absolute convergence of the series $\sum_{n=1}^{n=\infty} a_n$.

SECTION-B

Q.2)Attempt any three of the following. **[Marks 15]**

a) Prove that i) $A \rightarrow B \equiv\ \sim (A \wedge (\sim B))$ ii) $A \vee B \equiv\ \sim ((\sim A) \wedge (\sim B))$.

b) Show that the set of all ordered pairs of positive integers is countable.

c) Prove that convergent sequence of real numbers is bounded.

d) Find the limit superior and the limit inferior for the following sequences

i) 1 , 2 , 3 , 1 , 2 , 3 , 1 , 2 , 3 , ………………………

ii) $\left\{\sin \frac{n\pi}{2}\right\}_{n=1}^{n=\infty}$.

e) If $\sum_{n=1}^{n=\infty} a_n$ converges to A and $\sum_{n=1}^{n=\infty} b_n$ converges to B then show that $\sum_{n=1}^{n=\infty}(a_n + b_n)$ converges to A + B

SECTION-C

Q.3)Attempt any one of the following. **[Marks 10]**

a) i)Prove that Cauchy sequence of real numbers is convergent in $\mathbb{R}$.

ii) Show that the series$\sum_{i=1}^{n=\infty} \frac{1}{n}$ is divergent.

b) i) Prove that if $\{s_n\}_{n=1}^{n=\infty}$ converges to 1 then $\left\{\sqrt{s_n}\right\}_{n=1}^{n=\infty}$ converges to 1

ii) If $0 < x < 1$ then show that $\sum_{n=0}^{n=\infty} x^n$ converge to $\frac{1}{1-x}$.